THE SEARCH FOR EXTRA DIMENSIONS

— The Theory and Phenomena

Tao Han, Univ. of Wisconsin-Madison

(Univ. of Arizona, Oct. 29, 2004)

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⋆ Why and How ?

⋆ Gravitons and their Interaction with Matter

⋆ The Extra Dimensions: Flat, Warped, Universal

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♠. Phenomenological Implications

⋆ Table-top Gravity Experiments

⋆ Cosmological Constraints

⋆ Astrophysical Constraints

⋆ Collider Signatures

⋆ High-Energy Cosmic Neutrinos

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- ⋆ Why and How ?
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♠. Summary

♠. Old Stories about Extra Dimensions

1914: G. Nordstrom considered

$$d = 5 :$$

$$A_{\widehat{\mu}}(\widehat{\mu} = 0, 1, 2, 3, 5) \Rightarrow A_{\mu}(\mu = 0, 1, 2, 3) + \phi$$

with scalar ϕ identified as gravitational field!

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1915: A. Einstein told us:

$$g_{\mu\nu} \to \eta_{\mu\nu} + \kappa\ h_{\mu\nu}$$

metric $\text{dia}(\eta_{\mu\nu}) = (1, -1, -1, -1)$,

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1921: Th. Kaluza; 1926: O. Klein (+ others)

$$d = 5:$$

$$\gamma_{\widehat{\mu}\widehat{\nu}}(\widehat{\mu} = 0,1,2,3,5) \Rightarrow g_{\mu\nu}(\mu = 0,1,2,3) + A_\mu + \phi$$

leads to gravity+$E\&M$ in 4D.

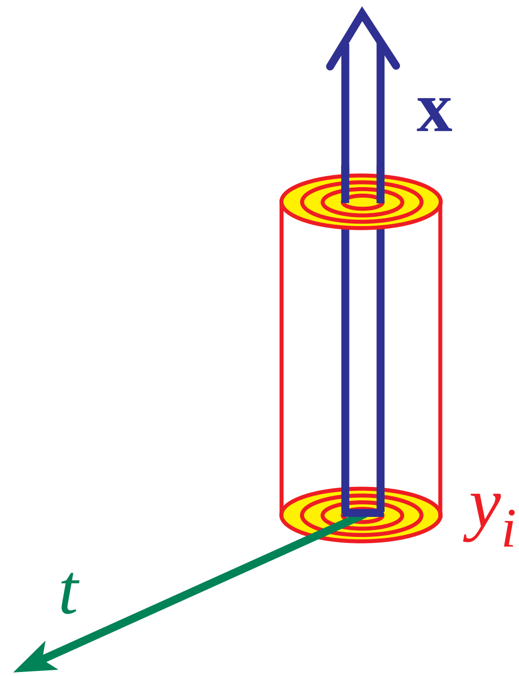
Wouldn't that be great !

(some quest with ϕ...)

What happened to the extra dimension y ?

- Too small to see ?

If our Universe $(\vec{x})$ is expanding as a function of t, why not some part (y) has shrunk or compactified ?



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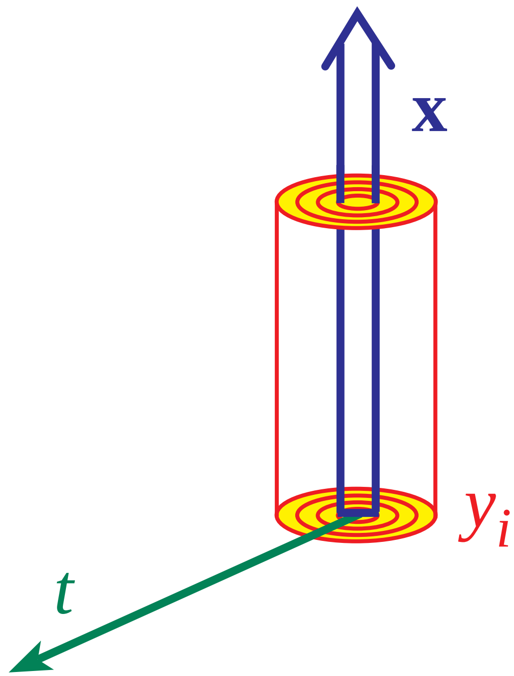

• Too elusive to probe ?

Our $E\&M$ probes can't get there ?

Only gravity lives there (possibly large).

If the extra dimension becomes compact (a circle of radius R), then all fields (gravitational, electromagnetic etc.) in y-dimension are periodic functions :

$$F(x,y) = \sum_{n=-\infty}^{\infty} F^n(x)\ e^{in\cdot y/R}.$$

Equation of motion:

$$(\partial^\mu\partial_\mu - \partial^y\partial_y)F(x,y) \Rightarrow (\partial^\mu\partial_\mu + \frac{n^2}{R^2})F^n(x)$$

$$\Rightarrow \quad m_n \sim \frac{n}{R} \quad \text{(a set of tower!)}$$

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$$\Delta M_{KK} = 1/R.$$

No γ_{KK}, e^-_{KK}, ... found $\Rightarrow$ R^{-1} large; or γ, e^- ... don't go there.

⋆ Extra Dimensions in String Theory:

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99–00's: Large extra dimensions ? ...
Arkani-Hamed, Antoniadis, Dimopoulos and Dvali: hierarchy
K. Dienes, E. Dudas, T. Gherghetta: coupling unification
Randall and Sundrum: hierarchy

♠ Large Extra Dimensions

⋆ Puzzle: the Mass Hierarchy Problem

- Weak scale by Fermi-weak interactions:

$$G_F = (\sqrt{2}\ v^2)^{-1} \approx 10^{-5}/\text{GeV}^2$$

$$\Rightarrow \quad v \approx 246 \text{ GeV}$$

In the Standard Model, all particle masses are proportional to v : $M_W = gv/2,\ m_e = g_e v/\sqrt{2},\ \ldots$

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† A mechanism to stabilize them: 10^{17} ?
Super-symmetry at $\mathcal{O}(100 - 1000 \text{ GeV})$

† Anything in between ?
Strong-electro-weak Unification at $\mathcal{O}(10^{17} \text{ GeV})$
Super-strings for quantum gravity ?

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Q: Is M_{pl} really fundamental ?

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Extrapolate to the Planck scale,

31 orders of magnitude down to $\ell_{PL} \sim 10^{-31}$ mm!

⋆ Beyond (1+3) Dimensions

Gauss' law says: $\oint \vec{E} \cdot d\vec{A} \propto M$, then in $d = 4 + n$:

$$F = G_N^{(d)} \frac{m_1 M}{r^{n+2}}.$$

After n-dim compactification on a radius R:

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Two fundamental scales: R and M_D,*
but not $M_{pl} \sim 10^{19}$ GeV !

* Arkani-Hamed, Antoniadis, Dimopoulos and Dvali.

Back to n-dim mass-scale relation, assuming $M_D \sim \mathcal{O}(M_S)$:

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If $M_S \ll M_{pl}$ as low as $\mathcal{O}(1 \text{ TeV})$, (**Good!**) then

$$R \sim \frac{M_{pl}^{2/n}}{M_S^{2/n+1}} \approx \begin{cases} \mathcal{O}(10^{14}\ mm) & \text{for } n = 1 \\ \mathcal{O}(0.1\ mm) & \text{for } n = 2 \\ \mathcal{O}(1.0\ fm) & \text{for } n = 7 \end{cases}$$

leads to "large" extra dimensions.

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Would We Have Seen it ? Not Necessarily:

- for $n=1$, $R \sim 10^8\ Km$, Impossible! Case ruled out.
- for $n \geq 2$, $R < 0.1\ mm$,

No direct astronomical/gravitational tests;

but can search for light KK states:

$m = n/R \sim 10^{-3}$ eV – 100 MeV.

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Horava-Witten's Picture:

"Standard Model fields only stick on the "3-brane",
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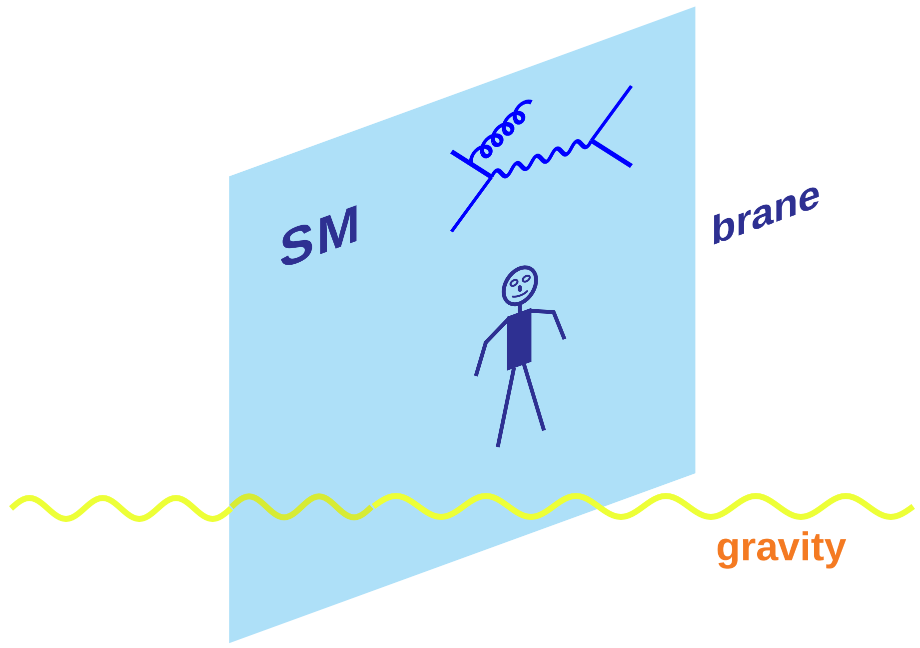
YOU are trapped on a "wall" (3-brane) !



Our world is a 4-dim. subspace (things with gauge interactions).
At least, gravitons propagate in extra-dim (probe the geometry).

⋆ Classification of KK Gravitons [‡]

In $d = 4 + n$ dimensions:

$$g_{\widehat{\mu}\widehat{\nu}} = \eta_{\widehat{\mu}\widehat{\nu}} + \widehat{\kappa} h_{\widehat{\mu}\widehat{\nu}}, \quad \widehat{\kappa}^2 = 16\pi G_N^{(d)},$$

where $h_{\widehat{\mu}\widehat{\nu}}$ is massless graviton field,
with $(n+4)(n+1)/2$ extra d.o.f.

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After compactification, there are:

1 spin-2 massive graviton
(n-1) spin-1 massive vectors
n(n-1)/2 spin-0 massive scalars

all mass-degenerate: $m^2 = \vec{n}^2/R^2$

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⋆ KK State Density

If R is large, there will be a high degeneracy:

$$\Delta \vec{n}^2 = \rho(m) dm^2,$$
$$\rho(m) = \frac{\pi^{n/2}}{\Gamma(n/2)} R^n m^{n-2}.$$

⇒ significant consequences!

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⋆ Graviton Interactions With SM Fields:

$$\int d^4x\sqrt{-\hat{g}}\ \mathcal{L} \approx -\frac{\kappa}{2}\int d^4x(h^{\mu\nu}_{\vec{n}} + \phi_{\vec{n}}\eta^{\mu\nu})\ T_{\mu\nu},$$

where $h^{\vec{n}}_{\mu\nu}$ is a graviton, $\phi_{\vec{n}}$ a scalar dilaton (specified by $\vec{n}$).

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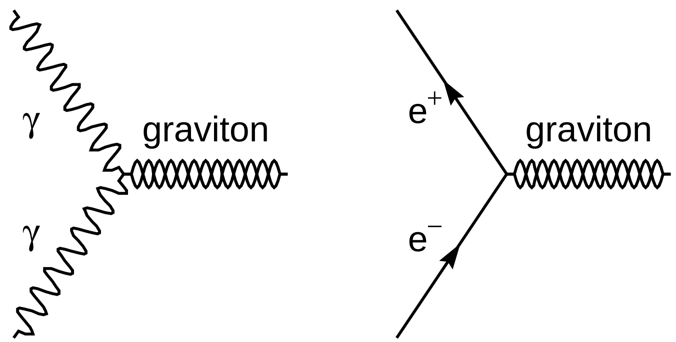
The energy-momentum tensor, $T_{\mu\nu}$, includes all matter:

$$T^{fermions}_{\mu\nu},\ T^{scalars}_{\mu\nu},\ T^{EW}_{\mu\nu},\ T^{QCD}_{\mu\nu}\ldots$$

The rule: a graviton couples to ANYTHING
(attach it on any legs or vertices in the SM)



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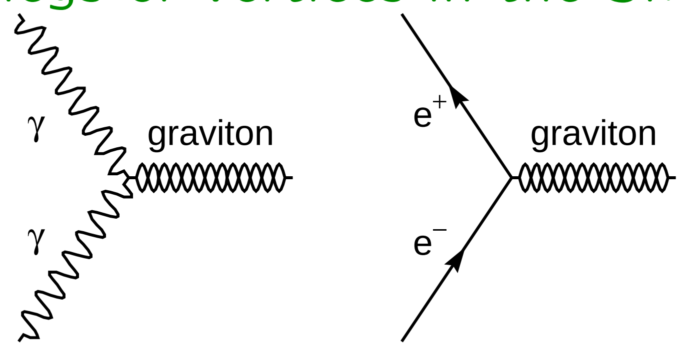

Although each graviton couples gravitationally,
the high-degeneracy leads to

$$\kappa^2\rho(m)dm^2 \sim \kappa^2 R^n m^{n-2} dm^2 \sim E^n/M_S^{n+2}$$

Effective coupling $\kappa^2 \sim \frac{1}{M_{pl}^2} \to \frac{1}{M_S^2}$!

⋆ General Consideration:

- In a factorizable flat metric:

$$ds^2 = \eta_{MN}\ dx^M dx^N, \quad M,N = (0,1,...,4+n),$$

with Minkowski metric $\eta_{MM} = (1,-1,-1,...)$.
In general, a $4+n$-dimensional gravity action:

$$S = \frac{1}{2} M_D^{n+2} \int d^{4+n}x\ \sqrt{-g}\mathcal{R},$$

where M_D: the $4+n$-dim Planck scale.

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- The most general Poincare invariant solution:

$$ds^2 = e^{2A(y)}\ \eta_{\mu\nu}\ dx^\mu dx^\nu - dy^i dy_i,$$

where $e^{2A(y)}$: a "warp" factor, which determines the shape of space-time in y.

The 4-dim Planck scale is:

$$M_{pl}^2 = M_D^{n+2}\ \int d^n y\ e^{2A(y)} \equiv M_D^{n+2}\ V_n.$$

Normally, M_{pl} depends on the shape in y.

⋆ The Randall-Sundrum Scenario

In a 5-dim space, Randall and Sundrum found a static solution:*

$$ds^2 = e^{2A(y)}\ \eta_{\mu\nu}\ dx^\mu dx^\nu - dy^2,$$

where the "warp" factor $A(y) = -ky,$

with k the curvature scale in the 5^{th}-dim.

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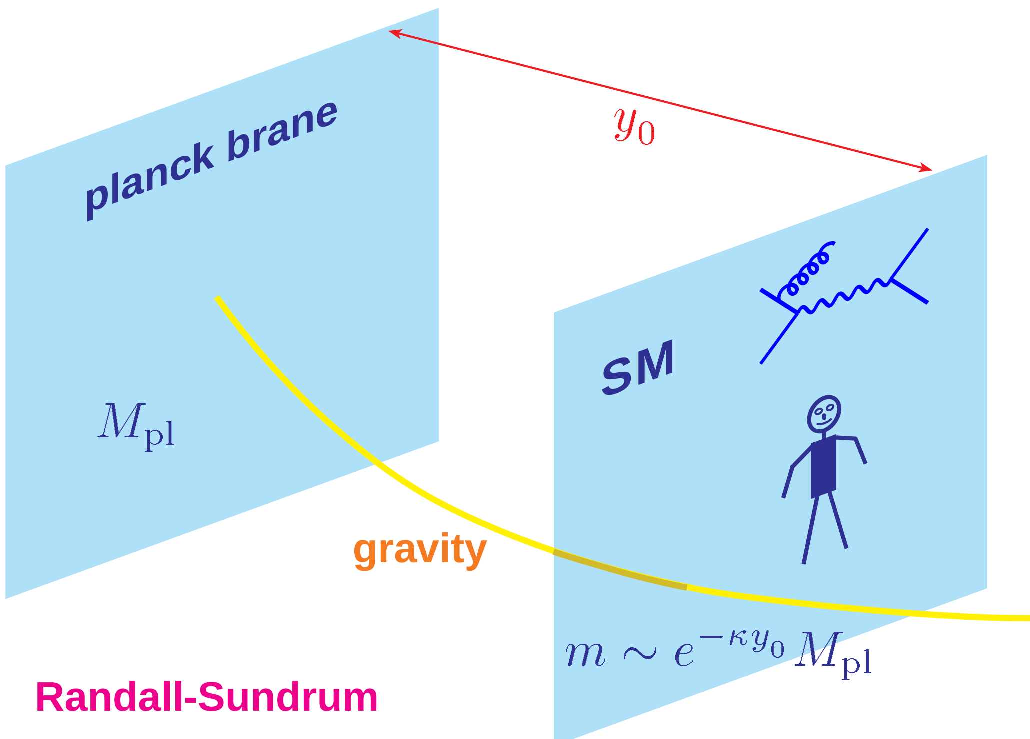
The extra dimension y is "warped".



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Features relevant to current interests:

- Mass hierarchy M_{pl}/M_{EW} generated on the two branes:

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To get $v \approx 246$ GeV, need $ky_0 \approx 40$.
the "size" of extra-dim: $y_0 \sim (10 - 100)\ l_{pl}$.

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- TeV KK resonances: $M_{KK} \sim e^{-ky_0} M_{pl}$:[‡]
  $G_{KK},\ g_{KK},\ A_{KK}, ..., f_{KK}...,$ with 1/TeV couplings.

[‡]Davoudiasl, Hewett, Rizzo, hep-ph/9909255.

⋆ Other Variations:

- "Universal extra dimensions" :

All standard-model particles propagate in the extra dimensions.
Then it must be compact*

$$M_{KK} > 1 \text{ TeV} \quad \text{or} \quad R > \frac{1}{1 \text{ TeV}} \sim 10^{-16} \text{ mm.}$$

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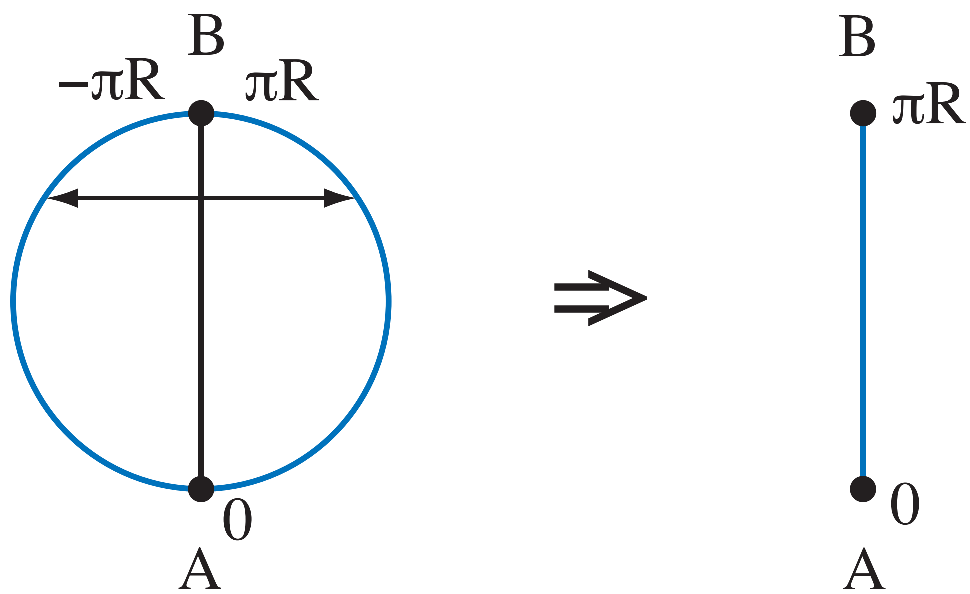
• "Orbifolding" S^1/Z_2:

a mapping symmetry and choice of boundary conditions.



*T. Appelquist, H.-C. Cheng, B.A. Dobrescu (2001).

⋆ Possible Outcomes of Extra Dimensions:

an exciting frontier for explorers

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- Newton's law: modified at both short and long distances.*
- EWSB: gauge-boson masses and the Higgs§, or "Higgsless".¶
- fermion masses: Yukawa couplings by displacement/overlapping‖
- ν masses/mixings: bulk neutrinos**
- GUTs: coupling consts. power-law running††
- SUSY GUTs: breaking by orbifolding*
- new cosmology;‡ cosmological const.†

*Dvali et al.

§Cheng et al.; Luty et al.; Hall et al.; Ignatius et al.; Z. Chacko and A. Nelson

¶C. Csaki et al.

‖Mirabelli and Schmaltz; Arkani-Hamed et al.

**Mohapatra, Nandi, Perez-Lorenzana;
Dienes et al.; Dimopoulos et al.

††Dienes, Dudas, Gherghetta; Dumitru and Nandi.

*Hall and Nomura; Hebecker and March-Russell et al.

‡Binetruy et al.; Kaloper et al.; Csaki et al.; Flanagan et al.; Cline et al.; Kanti et al.; Mohapatra et al.

†Arkani-Hamed et al.; Silverstein et al.; Luty et al.

♠. Phenomenological Implications

▷ At "low" energies

- "very low" : $E \ll 1/R,\ M_S$:

4−dim effective theory: Standard Model + weak classical gravity.

(as our present experimental knowledge.)

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▷ At "low" energies

- "very low" : $E \ll 1/R,\ M_S$:

4−dim effective theory: Standard Model + weak classical gravity.

(as our present experimental knowledge.)

- march into the extra-dimensions: $1/R < E \ll M_S$, $(4+n)$−dim world directly probed, and gravity effects observable:* mainly via light KK gravitons of mass

$$m_{KK} \sim 1/R,$$

or whatever propagate there ⇒ an effective theory (SM+KK).

*N. Arkani-Hamed, S. Dimopoulos, G. Dvali (1998);
G. Giudice, R. Rattazzi, J. Wells (1999);
T. Han, J. Lykken, R.J. Zhang. (1999);
Mirabelli, M. Peskin, M. Perelstein (1999);
J. Hewett (1999); T. Rizzo (1999); ...

▷ At intermediate energies $E \sim M_D,\ M_S$:
Stringy states significant* and resonances at the s-channel poles dominant:†

$$\mathcal{M}(s,t) \sim \frac{t}{s - M_n^2}, \quad M_n = \sqrt{n} M_S.$$

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▷ At "trans Planckian" energies $E > M_D, M_S$:
$(4+n)-$dim physics directly probed;
gravity dominant: black hole production*

$$M_{bh} = \sqrt{s} > M_D \text{ for } b < r_{bh}.$$

copiously produced at LHC or other TeV-scale experiments!

*G. Shui and H. Tye (1998); K. Benakli (1999).

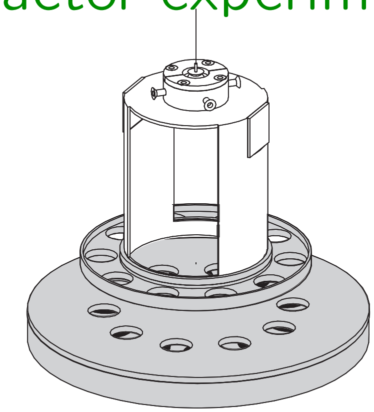
†Accomando, Antoniadis, Benakli (2000); Cullen, Perelstein, Peskin (2000).

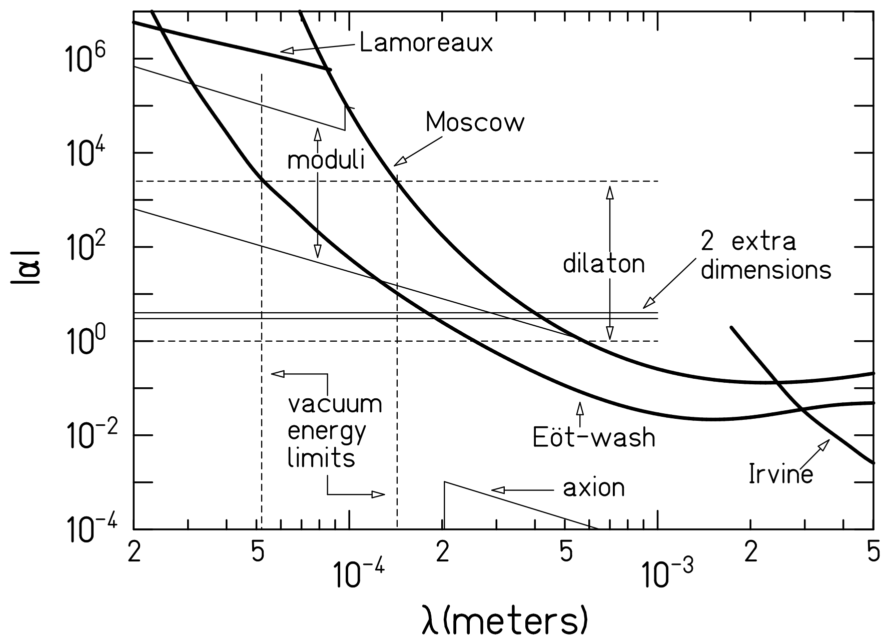
*T. Banks and W. Fischler (1999); E. Emparan et al. (2000);
S. Giddings and S. Thomas (2002);
S. Dimopoulos and G. Landsberg (2001).

⋆ Table-Top Gravity Experiments:

Univ. of Washington experimental report:†

A torsion pendulum/attractor experiment:

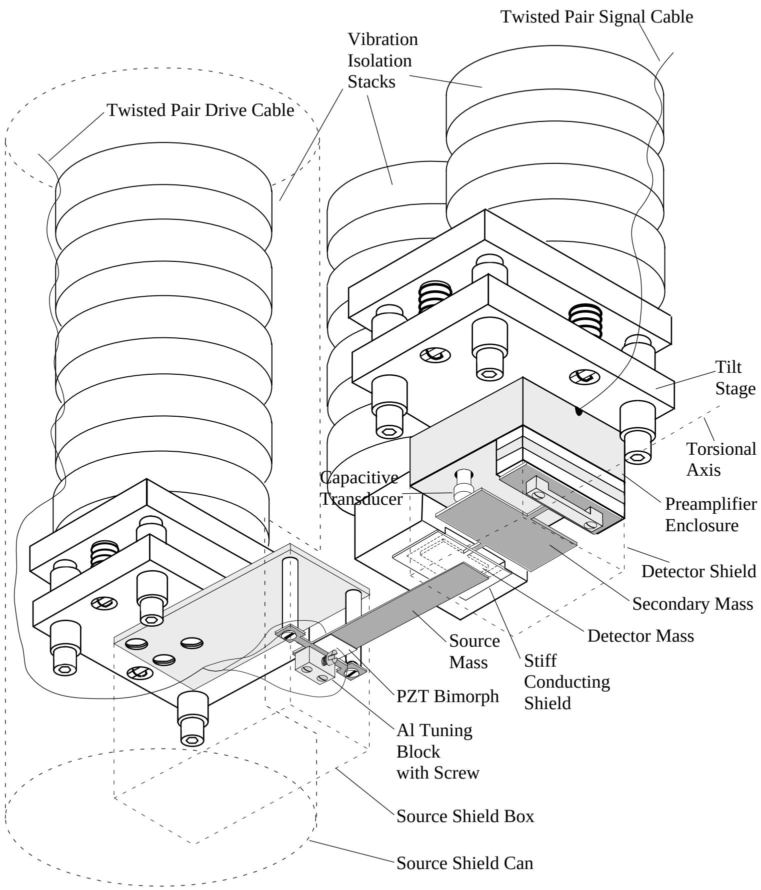


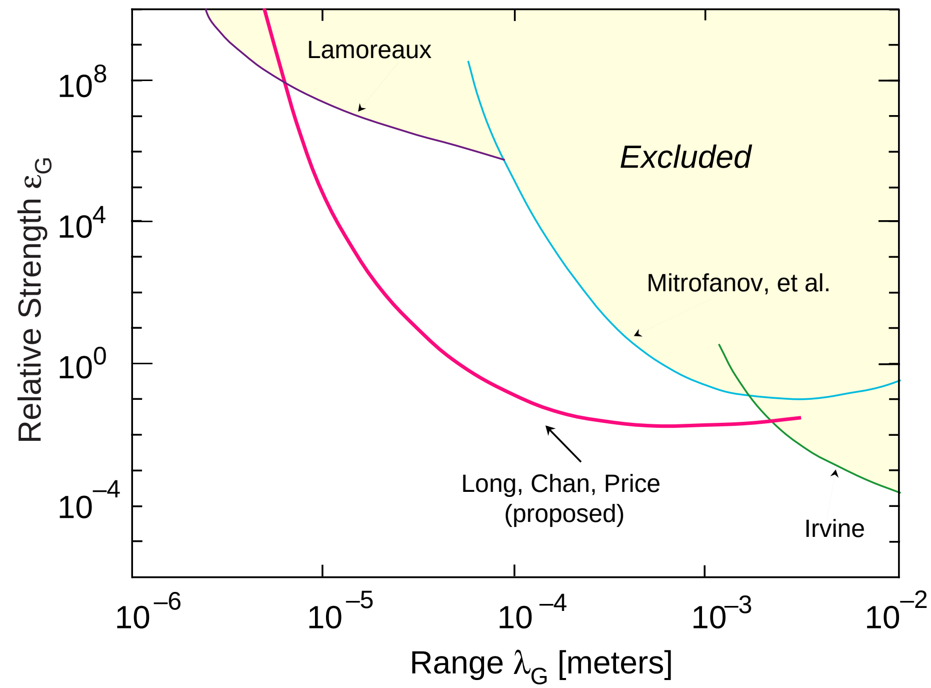


New force with $r >$ 0.2 mm excluded.

†E. Adelberger et al., Phys.Rev.Lett.86, 1418 (2001).

Univ. of Colorado experimental proposal:*





*Chan, Long, Price et al., `hep-ph/0009062`.

$\star$ Cosmological constraints: ADD

KK gravitons may be thermally produced in the early universe

$$\gamma\gamma,\ e^+e^-, \nu\bar{\nu} \to KK's,$$

but won't stay in equilibrium.

KK gravitons may decay to a pair of SM particles $(\gamma\gamma, \nu\bar{\nu}...)$,o but may be long-lived:

$$\tau(\tilde{h}) \approx \frac{10^3}{\kappa^2 m_{\tilde{h}}^3} \approx 6 \times 10^9\ yr\ (\frac{100 \text{ MeV}}{m_{\tilde{h}}})^3$$

[note $t_0 \approx 10^{10}$ years.]

Dangerous! if too much produced.

- not to overclose the Universe:*

$$\rho_{KK} < \rho_c = \frac{3H^2}{8\pi G_N} = 8.1 h^2 10^{-47} \text{ GeV}^4$$

$$\Rightarrow M_S > 6.5 \text{ TeV}, \text{ for n} = 2.$$

*L. Hall and D. Smith.

- BBN constraint:†

with $T_{reheat} > 1$ MeV

$\Rightarrow M_S > 10$ TeV, for $n = 2$.

- not to distort the observed cosmic photon spectrum:

$dn_\gamma/dE < 10^{-3}$ MeV^{-1}cm^{-2}s^{-1}ster^{-1}

$\Rightarrow M_S > 110$ TeV, for $n = 2$. (T_{reheat}-dependent)

If $M_S > 100$ TeV, then the model lost original motivation.

Thus

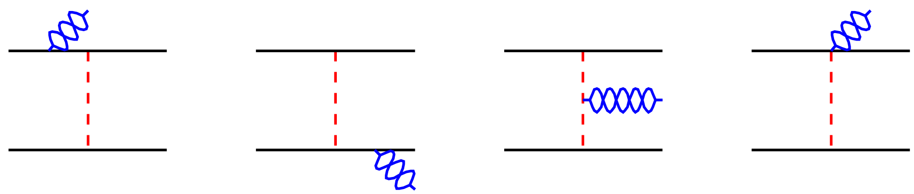
$n = 2$ is strongly disfavored.

†Arkani-Hamed et al.

$\star$ Supernova energy-loss: ADD*

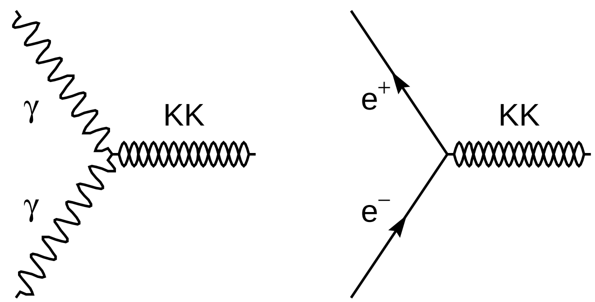
KK gravitons may take energy away from hot stars, the Sun, red giants and supernovae etc. rapidly via reactions:

$$N\ N \to N\ N + KK \text{ (missing energy) :}$$



and

$$\gamma\gamma,\ e^{+}e^{-} \to KK \text{ (missing energy) :}$$



Could have changed their evolution.

*S. Cullen and M. Perelstein;
Barger, Han, Kao and Zhang.

SN1987A energy-loss rate (mainly to $\nu' s$):

$$\epsilon_{SN} \approx 10^{19} \; erg \; g^{-1} \; s^{-1},$$

which leads to bounds ($T_{SN} \approx 20 - 60$ MeV):

$$n = 2: \quad M_S > 30 - 130 \text{ TeV (!)}$$
$$n = 3: \quad M_S > 2 - 9 \text{ TeV.}$$

The results have very little model-dependence.

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⋆ Neutron star heating:

Trapped KK gravitons in the SN core may overheat the NS.†

Observation of NS radiation ($KK \to \gamma\gamma,\ \ldots$) puts limit on

$$n = 2: \quad M_S > 100 \text{ TeV}$$
$$n = 3: \quad M_S > 10 \text{ TeV (!)}$$

The results have very little model-dependence.

$n \leq 3$ is strongly disfavored.

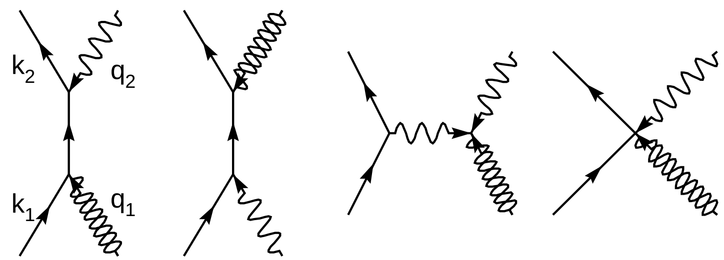
†S. Hannestad and G. Raffelt, `hep-ph/0110067`.

⋆ Collider Searches for Extra Dimensions:

A. Collider Signals I (ADD)

Real KK Emission: Missing Energy Signature*

a. $e^+e^- \rightarrow \gamma + KK$ (γ+missing energy)



n − dim :	at LEP2	at LC(500)
$n = 4$	$M_S >$ 730 (GeV)	4500
$n = 6$	$M_S >$ 520 (GeV)	3100

*Giudice, Rattazzi and Wells;
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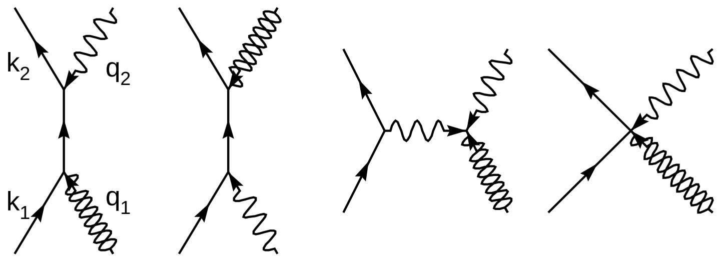

n − dim :	at LEP2	at LC(500)
$n = 4$	$M_S > 730$ (GeV)	4500
$n = 6$	$M_S > 520$ (GeV)	3100

b. $p\bar{p} \rightarrow jet + KK$ (mono-jet+missing energy)

n − dim :	at Tevatron	at LHC
$n = 4$	$M_S > 900$ (GeV)	3400
$n = 6$	$M_S > 810$ (GeV)	3300

*Giudice, Rattazzi and Wells;
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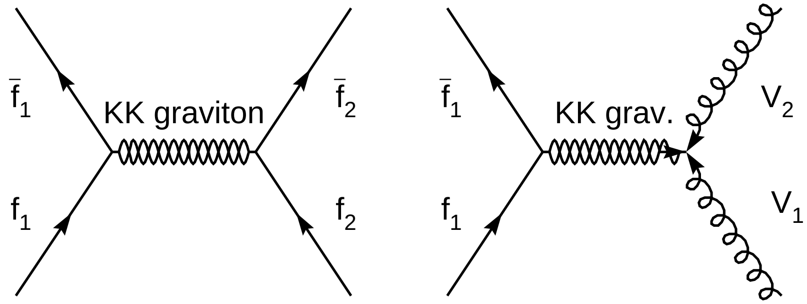
B. Collider Signals II (ADD)

Virtual KK Graviton Effects‡

On four-particle contact interactions:



Sum over virtual KK exchanges:

$$
\begin{aligned}
i\mathcal{M} \; &\sim \; \overline{f}\mathcal{O}_i f \; \overline{f}\mathcal{O}_j f \int_0^\infty \frac{dm_{\vec{n}}^2 \; \kappa^2 \rho(m_{\vec{n}})}{s - m_{\vec{n}}^2 + i\epsilon} \\
&\sim \; \frac{s^2}{M_S^4} \, \overline{f}\mathcal{O}_i f \; \overline{f}\mathcal{O}_j f.
\end{aligned}
$$

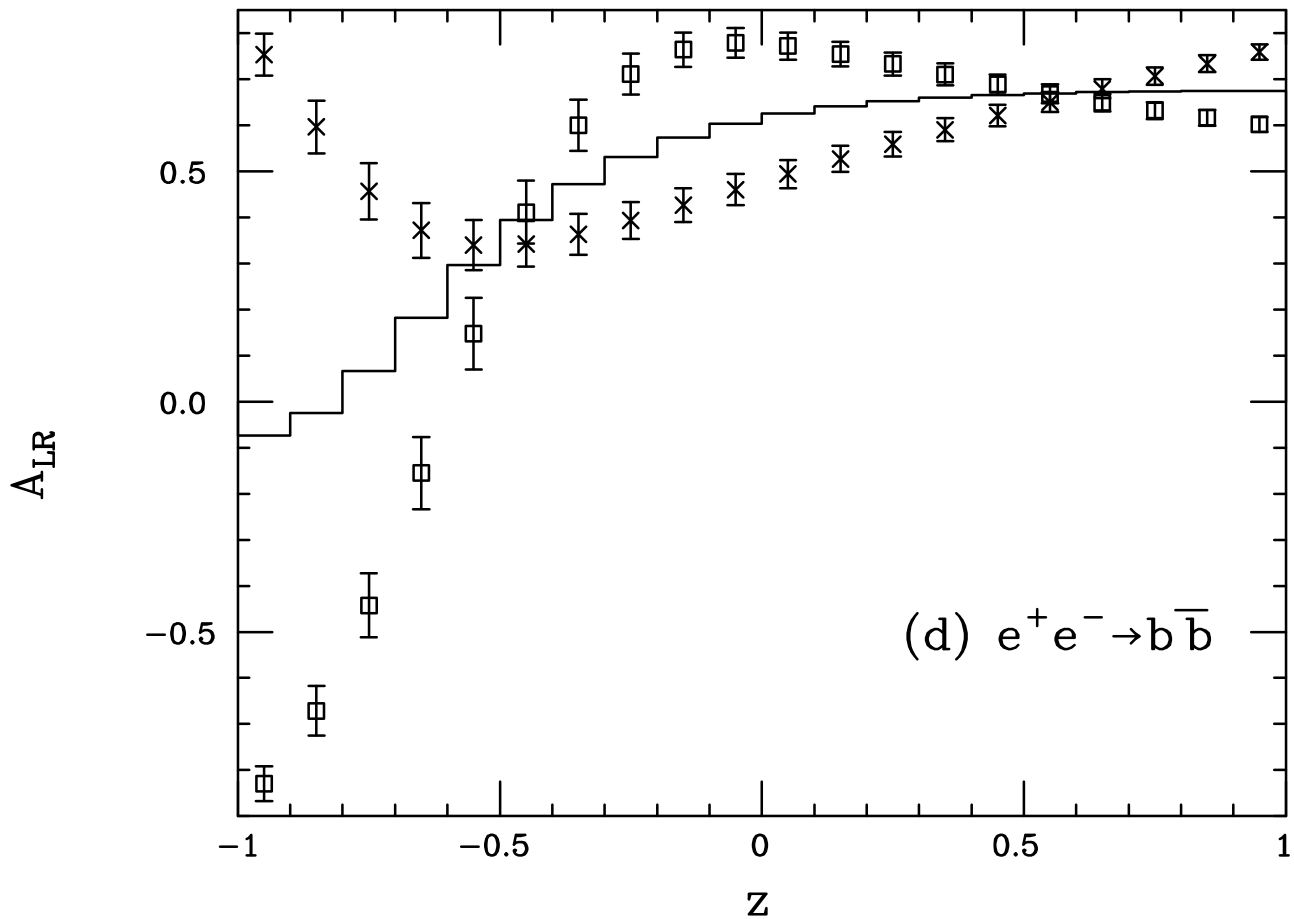
Again, effective coupling $\kappa^2 \sim \frac{1}{M_{pl}^2} \to \frac{1}{M_S^2}$!

‡ Hewett; Han, Lykken and Zhang; Rizzo; Cheung; Agashe and Deshpande; Nussinov and Shrock; Shiu, Shrock and Tye; Atwood, Bar-Shalom and Soni; Mathews, Raychaudhuri and Sridhar; ... ...

Collider sensitivities via virtual KK graviton exchanges: ADD

Reaction	LEP II (2 fb^{-1})	LC (100 fb^{-1})
$e^+e^- \to f\bar{f}$	1.15	$6.5\sqrt{s}$
$e^+e^- \to e^+e^-$	1.0	$6.2\sqrt{s}$
$e^-e^- \to e^-e^-$		$6.0\sqrt{s}$
$e^+e^- \to \gamma\gamma$	1.4	$3.2\sqrt{s}$
$e^+e^- \to WW/ZZ$	0.9	$5.5\sqrt{s}$
	Tevatron (2 fb^{-1})	LHC (100 fb^{-1})
$p\overset{(-)}{p} \to \ell^+\ell^-$	1.4	5.3
$p\overset{(-)}{p} \to t\bar{t}$	1.0	6.0
$p\overset{(-)}{p} \to jj$	1.0	9.0
$p\overset{(-)}{p} \to WW$	0.8	
$p\overset{(-)}{p} \to \gamma\gamma$	1.4	5.4
	$\gamma\gamma$ Collider (100 fb^{-1})	
$\gamma\gamma \to \ell^+\ell^-/t\bar{t}/jj$	$4\sqrt{s}$	
$\gamma\gamma \to \gamma\gamma/ZZ$	$(4-5)\sqrt{s}$	
$\gamma\gamma \to WW$	$11\sqrt{s}$	

Qualitative differences for signal/background distributions, due to the spin-2 exchange:



LR asymmetry for $e^+e^- \to b\bar{b}$ at $\sqrt{s} = 500$ GeV.
Solid: SM; “data” points for $\lambda = \pm 1$ with $= 75\ fb^{-1}$.

C. KK Resonant States at Colliders: (RS)

a. SM KK Particles:

If the SM fields (photons, electrons, Z, W, H^0...) also propagate in extra dimensions, then they have KK excitations.‡

Direct search bounds:

$$M^*_{\gamma,Z,W} \sim \frac{1}{R} > 4\ \text{TeV}.$$

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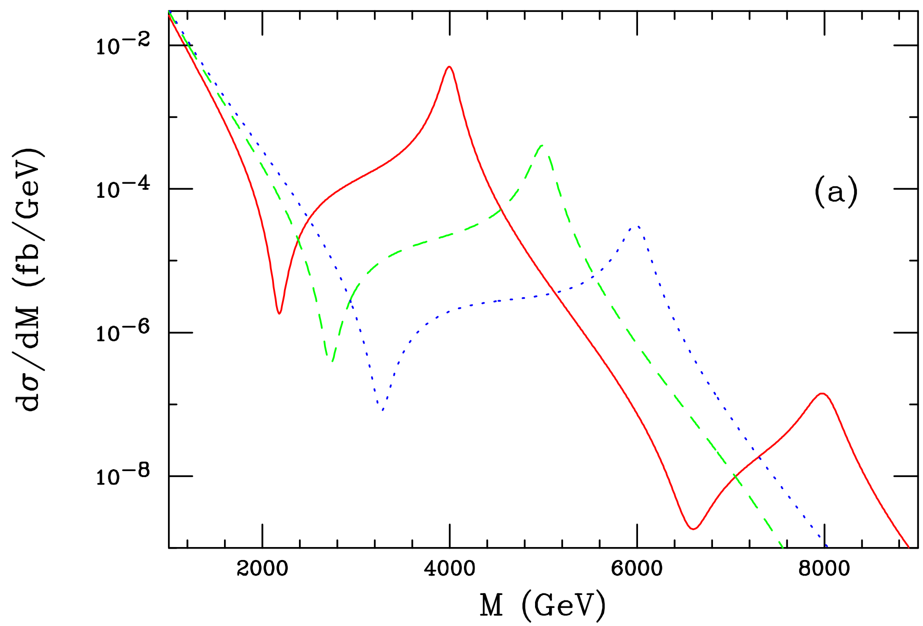
Resonant production at the LHC:



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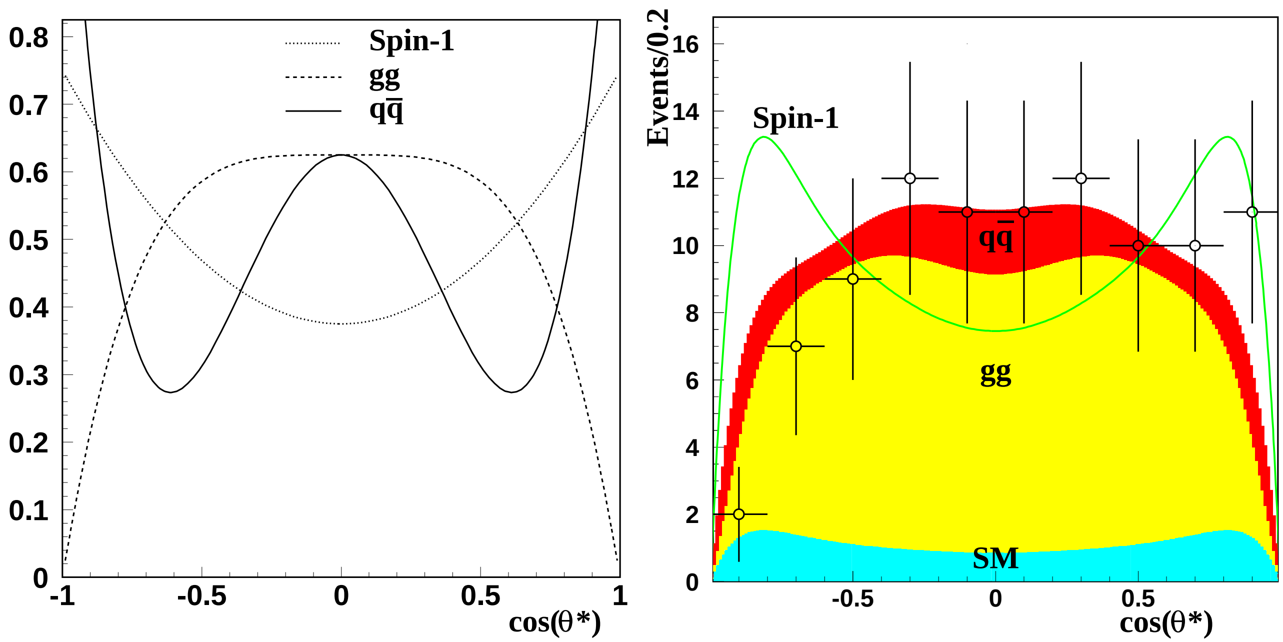
b. Heavy KK gravitons

DY $\ell^+\ell^-$ angular distributions:

$gg \to G \to e^+e^-$	$1 - \cos^4\theta^*$
$q\bar{q} \to G \to e^+e^-$	$1 - 3\cos^2\theta^* + 4\cos^4\theta^*$
$q\bar{q},\ gg \to V \to e^+e^-$	$1 + \alpha\cos^2\theta^*$
$q\bar{q},\ gg \to S \to e^+e^-$	1

At the LHC (ATLAS simulation*),



*Allanach et al., hep-ph/0006114

D. Stringy States at Colliders

Future colliders may reach the TeV string threshold thus directly produce the "stringy" resonant states.†

Amplitude factor near the resonance

$$\mathcal{M}(s,t) \sim \frac{t}{s - nM_S^2}, \quad \text{its mass } M_n = \sqrt{n} M_S.$$

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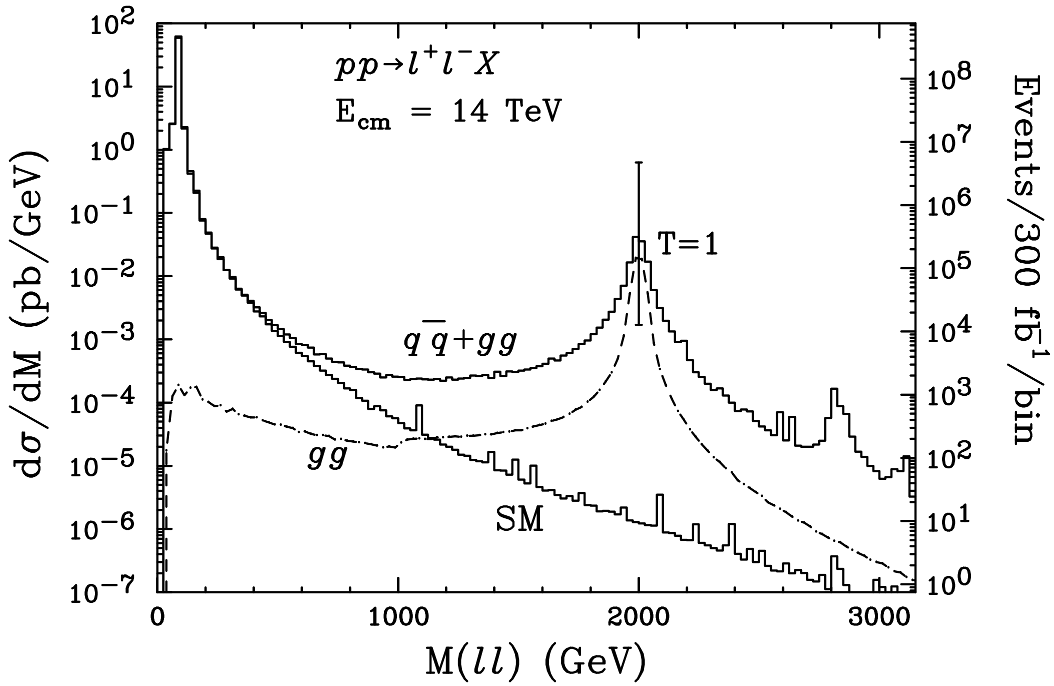

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E. Black Hole Production at Colliders*

For a black hole of mass M_{BH}, its size is

$$r_{bh} = \frac{1}{\sqrt{\pi} M_D} \left[\frac{M_{BH}}{M_D} \left(\frac{8\Gamma\left(\frac{n+3}{2}\right)}{n+2} \right) \right]^{\frac{1}{n+1}}$$

$$\approx \frac{1}{M_D} \left(\frac{M_{BH}}{M_D} \right)^{\frac{1}{n+1}} \rightarrow \frac{M_{BH}}{M_{pl}^2} \text{ in 4d.}$$

* Banks and Fishler, hep-th/9906038.

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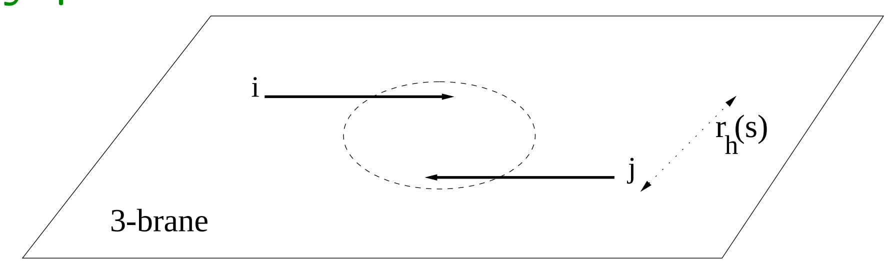
At higher energies and shorter distances (impact parameter)

$$E_{cm} > M_{BH} > M_D, \quad b_{impact} < r_{bh},$$

black holes formation is the dominant quantum gravity phenomena.



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Black holes copiously produced at the LHC energies,[†]

M_{BH}	$n = 4$	$n = 6$
5 TeV	1.6×10^5 fb	2.4×10^5 fb
7 TeV	6.1×10^3 fb	8.9×10^3 fb
10 TeV	6.9 fb	10 fb

[†] Giddings and Thomas, hep-ph/0106219; Dimopoulos and Landsberg, hep-ph/0106295; M.H. Reno, I. Sarcevic (2002).

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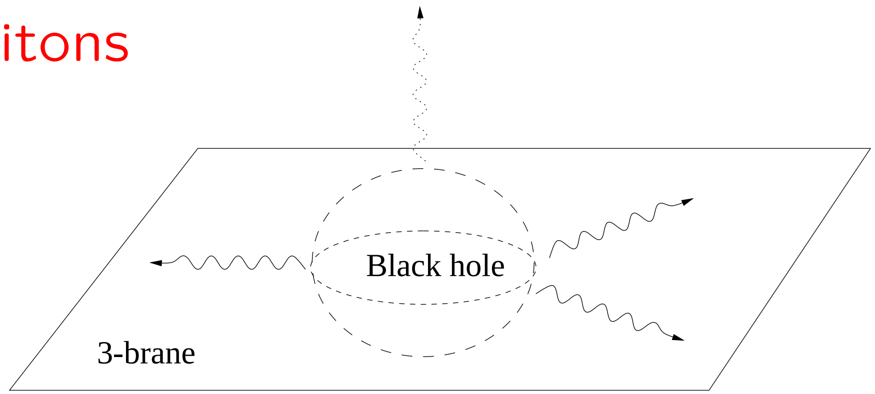
Black holes "decay" via Hawking radiation:†

$\gamma,\ \nu,\ e^{\pm},\ hadrons, \ldots\ W^{\pm},\ Z\ldots,$ gravitons



Spectacular events:

- very luminous in the detector!
- lepton-number/baryon-number violation;
- spherical (?) ... ...

† Giddings and Thomas, hep-ph/0106219; Dimopoulos and Landsberg, hep-ph/0106295; M.H. Reno, I. Sarcevic (2002).

† Emparan, Horowitz, Myers, hep-th/0003118.

⋆ High-Energy Cosmic Neutrinos:

- HE νN scattering well-understood in the SM *

$$\sigma(\nu_L N) = \int dx \sum_f \tilde{\sigma}(\nu_L f)\; x f(x, Q^2)$$

can be made use of to study physics beyond the SM:

$$Int.\ Length \sim \frac{1}{n_V \sigma_\nu}.$$

*R. Gandhi, C. Quigg, M.H. Reno, I. Sarcevic (1996, 1998);
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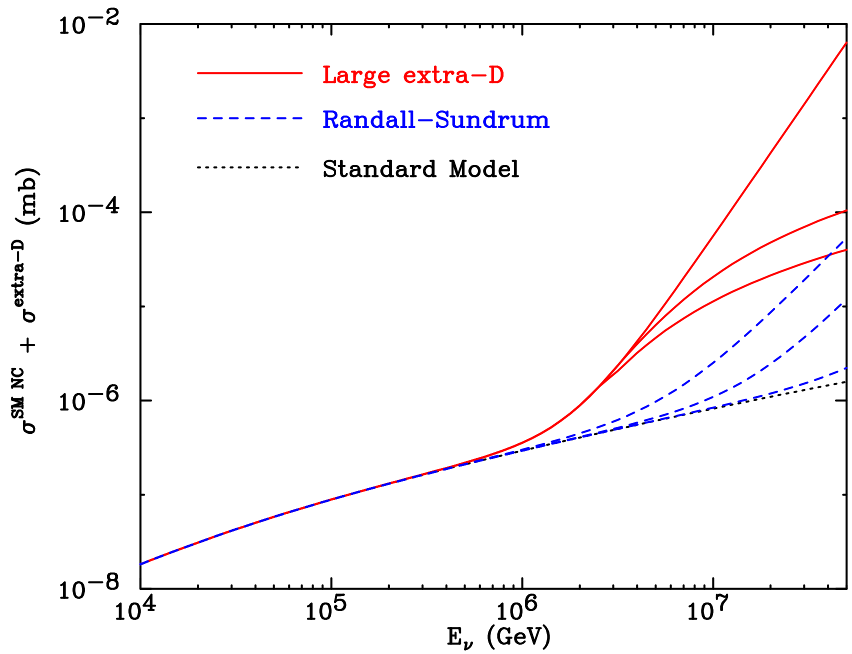

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HE Neutrino detectors

Auger air shower Observatory:
3000 km^2 fluorescence detector.
we look for quasi-horizontal air-shower events.

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Many studies on low-gravity scale models:[†] [‡]

Cosmic neutrino detectors have a good chance to observed quantum gravity/stringy effects if

$$M_S \sim 1 - 3 \text{ TeV}.$$

[†]Nussinov and Shrock (1999); Domokos and Kovesi-Domokos (1999); Jain *et al.*, (2000); Alvarez-Muniz *et al.*, (2002).

[‡]Feng and Shapere (2002); Emparan *et al.*, (2002); Ringwald and Tu (2002); Anchordoqui and Goldberg (2002); S. Dutta, M.H. Reno, I. Sarcevic (2002).

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  table-top gravity: if $\ell \sim 0.1 - 0.01$ mm;
  cosmic neutrinos: if $M_S \sim 1 - 3$ TeV; predate the LHC?
  colliders: if $M_S \sim 1 - 5$ TeV:
  production of KK gravitons, SM KK modes,
  stringy states and even black holes,
  or other unexpected ...

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Go Search for the Extra Dimensions !