832 Homework Set 1 (due Feb. 8)

Reflection of calculational technicality:

Problem 1: b-quark decay

The decay of a "beauty" quark (b) may be similar to β-decay, that can go to an electron (the β ray), an anti-neutrino, plus a charm quark (c):

$$b \to c + e^- + \bar{\nu}_e.$$

The invariant matrix element for this decay is given by Fermi's theory of weak interactions (that can now be understood in a more fundamental theory, the Standard Model)

$$i\mathcal{M} = -\frac{G_F V_{cb}}{\sqrt{2}}\ \overline{u_c}\gamma^\alpha(1-\gamma^5)u_b\ \overline{u_e}\gamma_\alpha(1-\gamma^5)v_{\bar{\nu}_e},$$

where $G_F = 1.16637 \times 10^{-5}/\mathrm{GeV}^2$ is the Fermi constant, V_{cb} the charm-beauty mixing element of the Cabibbbo-Kobayashi-Maskswa matrix.

(1). Neglect the masses of the final state particles $(e^-, \bar{\nu}_e, c)$, derive the partial decay width (30 pts.)

$$\Gamma(b \to ce\nu) = \frac{G_F^2 |V_{cb}|^2\ m_b^5}{192\pi^3}.$$

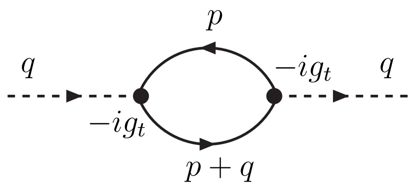
[Hint: You are supposed to find $|\mathcal{M}|^2 \propto (p_b \cdot p_\nu)(p_c \cdot p_e)$. Using the three-body phase space as in HW-11 last year, you may reduce the integration to $\int_0^1 dx_1 \int_{1-x_1}^1 dx_3$.]

(2). If we count all possible decay channels to obtain the total decay width, there will be approximately a factor of 9 ($W^{-*} \to e^-\nu_e,\ \mu^-\nu_\mu,\ \tau^-\nu_\tau,\ d\bar{u},\ s\bar{c}$). Take the b-quark mass to be $m_b = 5$ GeV and its lifetime $\tau_b = \Gamma_{tot}^{-1} = 1.6 \times 10^{-12}$ s, determine the value of $|V_{cb}|$. Just for fun, look up the muon (μ) lifetime and understand the difference. (20 pts.)

[Hint: Your result should read $|V_{cb}| \approx 0.025$, where the current experimental determination gives $|V_{cb}| \approx 0.04$.]

Problem 2: One-loop correction to the Higgs mass

Calculate the 1-loop correction to the Higgs mass $\Sigma(q^2)$ due to the Yukawa interaction of the top quark and Higgs field. For simplicity, give your result for $q^2 = 0$.



(1). Do the calculation with a cutoff Λ as a regulator in the momentum integration. (20 pts.)

[Hint: The result is of the form:

$$\Sigma(q^2 \to 0) = -\frac{3g_t^2}{4\pi^2}(\Lambda^2 - 3m_t^2 \ln \frac{\Lambda^2}{m_t^2}). \tag{1}$$

This is the famous "quadratic divergence" of the Higgs mass, indicating that a "natural theory" must have new physics beyond the Higgs and the top quark to show up below or near Λ.]

(2). Do the calculation with Pauli-Villars regularization by a covariant cutoff Λ. (20 pts.)

[Hint: You need a twice-subtraction (formally $\Lambda_1,\ \Lambda_2$) to regularize the quadratic divergence. At the end, you are supposed to find the same result as in Eq. (1).]

(3). Do the calculation with dimensional regularization in $n = 4 - 2\epsilon$, Compare with the previous results by identifying the divergent terms. (10 pts.)

Hint: You are supposed to interpret your result the same as in Eq. (1) with the identification as simple poles in $n = 2$ and $n = 4$ respectively

$$[1 - \epsilon(\gamma_E - \ln \frac{4\pi}{m_t})]\ \frac{m_t^2}{\epsilon - 1} \to \Lambda^2,$$
$$[1 - \epsilon(\gamma_E - \ln \frac{4\pi}{m_t})]\ \frac{1}{\epsilon} \to \ln \frac{\Lambda^2}{m_t^2}.$$

[Note: For details of the Yukawa theory, see p.118. Formulas on pages 190, 191, 193, 249, and 251 may be useful. There are three color states for a quark.]