Thanksgiving Homework set 11 (due Nov. 27)

Three-body phase space:

Following the general definition of the Lorentz invariant n-body phase space element, the 3-body phase space element can be written as

$$\int dPS_3 = \frac{d\vec{p}_1}{2E_1}\,\frac{d\vec{p}_2}{2E_2}\,\frac{d\vec{p}_3}{2E_3}\,\delta^4(P - p_1 - p_2 - p_3),$$

where p_1, p_2, p_3 are the momenta for final state particles under consideration; P is the total initial state momentum.

(1). Consider for simplicity a $1 \to 3$ decay process. Work in the c.m. frame, $\vec{P} = 0,\ P_0 = \sqrt{s}$. Show that the phase space element can be expressed as

$$\int dPS_3 = \frac{1}{8} dE_1\ dE_3\ d\Omega_1\ d\phi_3,$$

where $d\Omega_1$ is the solid angle for $\vec{p}_1$ and $d\phi_3$ the azimuthal angle for $\vec{p}_3$ about $\vec{p}_1$.
[To handle the factor δ^4, you need to integrate out $d\vec{p}_2$ first. Then make use of the momentum conservation relation $E_2^2 = (\vec{p}_1 + \vec{p}_3)^2 + m_2^2$ to get $E_2 dE_2 = p_1 p_3\ d\cos\theta_{13}$.]

(2). Integrating the angle variables out and scale the energy variables by $x_i = 2E_i/\sqrt{s}$ for $i = 1, 2, 3$, show that

$$\int dPS_3 = \frac{\pi^2}{4} s\ dx_1\ dx_3.$$

(3). For the simplest case $m_i = 0$ for $i = 1, 2, 3$, determine the physical region in the x_1-x_3 plane (the integration limits for x_1 and x_3). Historically, the event distribution in the energy plane E_i-E_j is called the Dalitz plot.

(4). Plot the single variable differential distribution for dPS_3/dE_1 in the parent rest frame for given masses $M = P_0 = \sqrt{s},\ m_i = 0$.