831 Homework set 12 (due Dec. 1)

Scattering matrix element and Feynman rules:

Given QED interactions

$$\begin{aligned} \mathcal{L}(x) &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m_e)\psi + (D^\mu\phi)^*(D_\mu\phi) - m_\phi^2\phi^*\phi, \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \quad D_\mu = \partial_\mu + ieA_\mu, \end{aligned}$$

where $\psi(x)$ is the electron field operator that annihilates an electron and creates a position, and $\phi(x)$ is a charged scalar field operator that annihilates a charged scalar H^+ and creates an H^-.

(1). Write down the interaction Lagrangian and Hamiltonian densities $\mathcal{L}_I(x),\ \mathcal{H}_I(x)$.

(2). Starting from the transition matrix element $i\mathcal{T}_{fi}$ in second order perturbative expansion, using the Wick's theorem, derive the covariant matrix element $i\mathcal{M}$ for the process

$$e^-(p_-) + e^+(p_+) \to H^-(q_-) + H^+(q_+). \tag{1}$$

(3). Write down the QED Feynman rules for the electron and charged scalar. Draw the Feynman diagram for the process (1) in the momentum space, and construct the scattering amplitude $i\mathcal{M}$.