731 Homework set 2 (due Sept.23)

1. Functions of operators:
For an operator Ω, one can define a function operator $f(\Omega)$, just as a regular function $f(x)$. The general prescription is

$$f(x) = \sum_{n=0}^{\infty} a_n x^n \quad \Longrightarrow \quad f(\Omega) = \sum_{n=0}^{\infty} a_n \Omega^n,$$

so that if $\Omega|\omega\rangle = \omega|\omega\rangle$, then $f(\Omega)|\omega\rangle = f(\omega)|\omega\rangle$.
For example,

$$e^{\Omega} = \sum_{n=0}^{\infty} \frac{1}{n!}\,\Omega^n,$$

and then $e^{\Omega}|\omega\rangle = e^{\omega}|\omega\rangle$.

Show that if H is a Hermitian operator, then
(1) $U \equiv \exp(iH)$ is a unitary operator (10 points).

(2) the determinant can be obtained by det $U = \exp(iTrH)$, where the trace can be written as the sum of the eigenvalues $TrH = \sum_j^n h_j$ (10 points).
(Hint: Recall that a Hermitian operator has real eigenvalues, and can be diagonalized by a unitary transformation. So it is convenient to work with the diagonal form of H.)

2. The Dirac notation: Sakurai's p.60, Problem 4 (20 points).

3. The Dirac notation and matrix elements: Sakurai's p.61, Problem 5 (20 points).