831 Homework set 2 (due Sept. 22)

Problem 1

In 1928, P. Jordan and E.P. Wigner proposed the “second quantization” for electron wave functions by introducing the “anticommutation relations”

$$\{b_r, b^\dagger_{r'}\} = \delta_{rr'}, \qquad \{b_r, b_{r'}\} = \{b^\dagger_r, b^\dagger_{r'}\} = 0, \tag{1}$$

where $b^\dagger_r$ and b_r are the creation and annihilation operators:

$$b^\dagger_r |0 >= |1_r > \qquad \text{and} \qquad b_r|0 >= 0, \tag{2}$$

with r denoting the relevant quantum numbers for a given particle. Show that this formulation satisfies the Pauli exclusion principle. You can introduce the occupation number operator $N_r = b^\dagger_r b_r$, if desirable.

Problem 2

In the Bardeen-Cooper-Schriefer theory of superconductivity, the annihilation and creation operators for a correlated pair of electrons are defined by

$$c_k = b_{-k\downarrow} b_{k\uparrow}, \qquad c^\dagger_k = b^\dagger_{k\uparrow} b^\dagger_{-k\downarrow}, \tag{3}$$

where b_{ks} and $b^\dagger_{ks}$ are expected to satisfy Eq. (1) in Problem I. Evaluate the commutators $[c_k, c_{k'}]$, $[c^\dagger_k, c^\dagger_{k'}]$ and $[c_k, c^\dagger_{k'}]$.