

831 Homework set 4 (due Oct.6)

1. The Pauli-Lubanski four-vector is defined in terms of the Poincare group generators as $W^\lambda = \frac{1}{2}\epsilon^{\lambda\mu\nu\sigma} J_{\mu\nu} P_\sigma$. Prove the following commutation relations

$$[W^\lambda, J^{\mu\nu}] = i(W^\mu g^{\nu\lambda} - W^\nu g^{\mu\lambda}), \quad [W^\lambda, W^\sigma] = i\epsilon^{\lambda\sigma\mu\nu} W^\mu P_\nu. \quad (1)$$

2. The classical Lagrangian density of a spin 1/2 fermion with a mass m is given by

$$\mathcal{L} = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x), \quad (2)$$

where ψ should be viewed as a 4-component Dirac spinor.

(a). Using Euler-Lagrange equation, derive the equation of motion for the fermion.

(b). The Lagrangian is invariant under a global gauge transformation $\psi \rightarrow e^{-i\alpha}\psi$. Derive the Noether current corresponding to this transformation, and show that the current is conserved.