731 Homework set 5 (due Oct.14, 2005)

1. Time-evolution of a Gaussian wave packet
Consider a one-dimensional Gaussian wave packet at $t = 0$,

$$\langle x'|\alpha\rangle = \frac{1}{(\pi d^2)^{1/4}} \exp\{\frac{i}{\hbar} p_0 x' - \frac{x'^2}{2d^2}\}.$$

Assume the wave packet is in a free motion, $H = p^2/2m$ (thus the energy eigenfunction is the same as the momentum eigenfunction).

(a). Determine the state evolution to a later time t, $\langle x'|\alpha, t\rangle$. (20 points)
Hint: You may make use of the result either in Eq. (1.7.42) of p. 58 (if still finding the integration messy, consider a variable shift $p - p_0 \to p$), or in Eq. (2.5.16) of p. 112.

(b). Calculate the probability density in the coordinate space (x') at the time t, $|\langle x'|\alpha, t\rangle|^2$. Discuss its evolution in t and its motion in x'. (10 points)

(c). (10 points) Consider d to be the typical size of the Gaussian wave packet at $t = 0$. Find numerical answers for the time t at which the wave packet of
(i) an electron of mass 9.11×10^{-31}kg initially localized in $d = 10^{-8}$ cm becomes 100 times wider;
(ii) a classical particle of mass 1g initially within $d = 0.1$ cm becomes 1% wider.
Comment on your numbers.

2. Sakurai's p.144, Problem 6 (20 points).
Hint: You may need the commutator relations in Eqs. (1.6.50) and (2.2.23).