831 Homework set 5 (due Oct.13)

1. Classical field of E & M:
Peskin-Schroeder: Problem 2.1 on page 33.

2. Quantum Klein-Gordon field:
Consider a free quantum Klein-Gordon field $\phi(x)$. The four-momentum operator is found to be

$$P^\mu = \int \frac{d^3p}{(2\pi)^3}\, p^\mu\, a_p^\dagger a_p$$

where p^μ is an eigenvalue of P^μ.
(a). Using the canonical commutation relations, show that

$$[P^\mu, a_k] = -k^\mu a_k, \tag{1}$$
$$e^{iP\cdot x} a_k e^{-iP\cdot x} = a_k e^{-ik\cdot x}, \tag{2}$$
$$[P^\mu, \phi(x)] = -i\partial^\mu \phi(x). \tag{3}$$

(Note: $e^A B e^{-A} = B + [A,B] + \frac{1}{2}[A,[A,B]] + \frac{1}{3}[A,[A,[A,B]]] + ...$)

(b). Let $|K\rangle$ be an eigenstate of P^μ, such that $P^\mu|K\rangle = K^\mu|K\rangle$. Show that a two-point correlation function under this state is translationally invariant

$$\langle K|\phi(x)\phi(y)|K\rangle = \langle K|\phi(x-y)\phi(0)|K\rangle. \tag{4}$$