731 Homework set 6 (due Oct. 28)

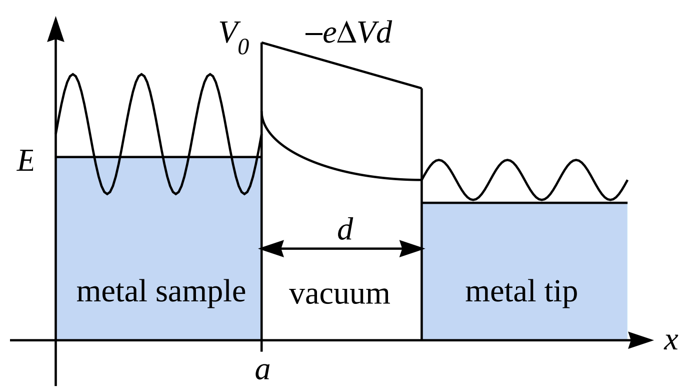
Semi-classical approximation: WKB Method

1. Bound state problem (20 points)

Using the WKB approximation, determine the energy eigenvalues for a one-dimensional simple harmonic oscillator.

2. STM (40 points)

The valence electrons in the metals are freely moving with energy E. There is a potential energy barrier $V_0 > E$ at the surface to prevent electrons from escaping. The energy difference $\Phi = V_0 - E$ is often called the "workfunction". Classically, the electrons will remain inside the metal due to the potential well. Quantum mechanically, the electrons can tunnel through the potential barrier to escape with certain probability. If a metal tip is introduced very close to the metal surface, it is possible to observe an electric current between the metal surface and the tip as a result of the tunneling effect. This is the principle for the STM (Scanning Tunneling Microscope, Nobel Prize in 1986).



Assume that the metal sample surface and the tip is separated by a distance d in vacuum. A small static electric potential difference ΔV is introduced between them to guide the tunneling current, as shown in the figure. The potential energy for the electrons is thus given by

$$V(x) = \begin{cases} 0 & \text{for } x < a \\ V_0 - e\Delta V(x-a)/d & \text{for } a < x < a+d \\ 0 & \text{for } a+d < x \end{cases}$$

where e is the magnitude of the electron's electric charge.

(1) Using the WKB method, calculate the transition probability T for the tunneling between the metal surface and the tip. (15 pts)

(2) Apart from the classical turning points, derive the condition for the WKB method to be valid. (5 pts) [Neglect the x-dependent piece at some point, and examine $e\Delta V/\Phi$.]

(3) Assume $\Phi \gg e\Delta V$, which leads to a square-well potential, show that the transition probability simplifies to

$$T \approx \exp(-\frac{2d}{\hbar}\sqrt{2m\Phi}). \tag{1}$$

Note that the exact solution exists for a square potential-well tunneling, as given in the Appendix, Eq. (A.3.4). What is the condition to obtain the exponential factor Eq. (1) by examining Eq. (A.3.4)? (10 pts)

(4) Assume that the workfunction $\Phi = 4$ eV, and the electron mass $m \approx 0.5$ MeV. what are the transition probabilities for $d = 1$ and 2Å? What do these numbers mean to you? (10 pts) [In doing numerics, life could be made easier to adopt the natural units: $\hbar = 1$, and $\hbar c = 1 \approx 2000$ eV·Å.]