835 Homework set VI (due Nov. 30)

You need to work out one of the three problems to get full credits. You should choose the suitably challenging ones for your own sake. You are of course encouraged to work out as much as you can.

Level I Problems:
(1) Define the transverse momentum of an outgoing electron to be

$$p_{eT} = p_e \sin\theta^*, \tag{1}$$

where θ^* is the polar angle in the partonic c.m. frame. Show that the partonic level differential cross section can be written as

$$\frac{d\hat{\sigma}}{dp_{eT}} = \frac{4p_{eT}}{s\sqrt{1-4p_{eT}^2/s}}\,\frac{d\hat{\sigma}}{d\cos\theta^*}. \tag{2}$$

For a resonant production, say the Drell-Yan process $q\bar{q} \to Z \to e^+e^-$, combining with a Breit-Wigner resonance, further show that

$$\frac{d\hat{\sigma}}{dm_{ee}^2\;dp_{eT}^2} \propto \frac{\Gamma_Z M_Z}{(m_{ee}^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}\,\frac{1}{m_{ee}^2\sqrt{1-4p_{eT}^2/m_{ee}^2}}\,\frac{d\hat{\sigma}}{d\cos\theta^*}. \tag{3}$$

The integrand appears to be singular at $p_{eT}^2 = m_{ee}^2/4$. Is it reasonable or harmful? The enhancement in the mass distribution near $p_{eT} = M_Z/2$ is called the Jacobian peak.

(2) For a massless quark (q), derive the color/spin sumed and averaged matrix element squared for the gluon fusion $gg \to q\bar{q}$, in terms of the Mandelstam variables.
(You can check the answer in the text and reference books.)

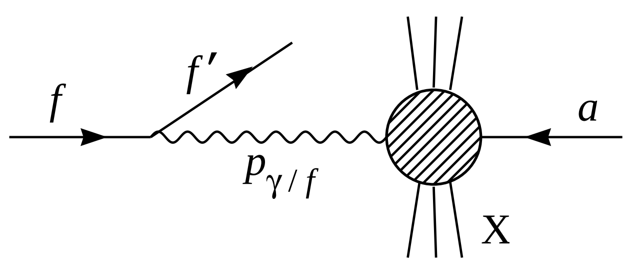
Level II Problems:

(1) Derive the color/spin sumed and averaged matrix element squared for gluon scattering $gg \to gg$, in terms of the Mandelstam variables.
(Note that there are 4 diagrams at the tree level. You can check the answer in the text and reference books.)

(2) `Weizsäcker-Williams approximation:` (also known as the effective photon approximation.)
Photon beams may be obtained by collinear radiation off charged particles. A reaction $e^- a \to e^- X$ can be expressed by the dominant photon subprocess $\gamma a \to X$,

$$\sigma(e^- a \to e^- X) \approx \int dx\ P_{\gamma/e}(x)\ \sigma(\gamma a \to X).$$

known as the Weizsäcker-Williams approximation, as depicted in the figure below. This approximation significantly simplifies the matrix element calculations.



For an electron of energy E, show that the probability of finding a collinear photon of energy xE is given by

$$P_{\gamma/e}(x) = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\ln\frac{E^2}{m_e^2}.$$

Also comment on the physical meaning of the terms $1/x$ and $\ln m_e^2$.
In deriving this formula, you need to assume an on-shell real photon.
(The kernel of this expression is the same as $q \to g$ splitting function.)

Level III Problems:

(1) Same as (2) in Level II.

(2) `Effective` W`-boson approximation:` Extend the above calculation to a massive gauge boson ($V = W^{\pm}, Z^0$), the effective W-boson approximation (EWA) states that the scattering cross section of a fermion f with energy $E \gg M_V$ off a target a is given by

$$\sigma(fa \to f'X) \approx \int dx\ dp_T^2\ P_{V/f}(x, p_T^2)\ \sigma(Va \to X),$$

where the probability of finding a (nearly) collinear gauge boson V of energy xE and transverse momentum p_T (with respect to $\vec{p}_f$) is approximated by

$$\begin{aligned} P^T_{V/f}(x, p_T^2) &= \frac{g_V^2 + g_A^2}{8\pi^2}\ \frac{1 + (1-x)^2}{x}\ \frac{p_T^2}{(p_T^2 + (1-x)M_V^2)^2}, \\ P^L_{V/f}(x, p_T^2) &= \frac{g_V^2 + g_A^2}{4\pi^2}\ \frac{1 - x}{x}\ \frac{(1-x)M_V^2}{(p_T^2 + (1-x)M_V^2)^2}, \end{aligned}$$

where T (L) denotes the transverse (longitudinal) polarization of the massive gauge boson.

Compare the p_T distributions for $T,\ L$ polarizations in the low-p_T^2 ($\ll M_V^2$) and high-p_T^2 ($\gg M_V^2$) regions, and think (or ask me) about the physical implications.

You may consult with the references: G. Kane et al., Phys.Lett.B148, 367 (1984); and S. Dawson, Nucl.Phys.B249, 42 (1985).