731 Homework set 7 (due Nov. 4)

The propagator

1. A particle is confined in a one-dimensional rigid-well potential in $0 \le x \le L$, see the appendix for stationary solutions.

(a). Calculate the propagator or the transition amplitude explicitly in terms of the energy eigenfunctions (10 pts)

$$K(x,t;\ x',0) = \langle x,t|x',0\rangle. \tag{1}$$

(b). If the wave function of the initial state is arranged like a poked string,

$$\psi(x,0) = \begin{cases} cx & \text{for } 0 \le x \le L/2 \\ c(L-x) & \text{for } L/2 \le x \le L, \end{cases}$$

where c is a normalization constant (you don't need to determine it), find the wave function at a later time $\psi(x,t)$. (15 pts)

(c). If the system is a classical string, it satisfies the wave equation

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2},$$

instead of the Schrödinger equation. If the wave function of the initial state is arranged like a poked string as above and no initial motion (why do I give this?), do you think that the above propagator approach should work and if so what do you expect to change in the result of evaluating the wave function evolution at the time t? (10 pts)

2. The propagator in momentum space is written as $K(p_2,t_2;\ p_1,t_1)$, analogous to Eqs. (2.5.7) and (2.5.8). Show that it is also equal to the transition amplitude $\langle p_2,t_2|p_1,t_1\rangle$ like Eq. (2.5.26). Give an explicit expression for the free particle case. (15 pts)