

831 Homework set 8 (due Nov. 3)

Parity, Time reversal, and charge conjugation

(1). Consider the anti-symmetric tensor of a fermion bilinear $\bar{\psi}\sigma^{\mu\nu}\psi$ with $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. Show its transformation properties under P, T, C and CP , and $\Theta = CPT$.

(2). Consider a quantum complex Klein-Gordon field $\phi(x)$. The free field operators can be written as

$$\begin{aligned}\phi(x) &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} (a_p e^{-ip \cdot x} + b_p^\dagger e^{ip \cdot x}), \\ \phi^*(x) &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} (a_p^\dagger e^{ip \cdot x} + b_p e^{-ip \cdot x}).\end{aligned}$$

(a). Adopting appropriate transformation properties of the creation and annihilation operators under P, T, C , derive the field transformation

$$P\phi(x)P^\dagger, \quad T\phi(x)T^\dagger, \quad C\phi(x)C^\dagger.$$

(b). Define a current

$$j^\mu(x) = i(\phi^* \partial^\mu \phi - \partial^\mu \phi^* \phi).$$

Show its transformation properties under P, T and C .