831 Syllabus (Fall, 2006): Quantum Field Theory I

1 Introduction to Quantum Field Theory

1.1 Foundations of Modern Physics

- Quantum mechanics, Special relativity, Natural units

1.2 Relativistic Quantum Mechanics

a. The Klein-Gordon equation

b. The Dirac equation

c. Relativistic wave equations for higher spin states

1.3 Necessity of the Field Viewpoint

a. Electromagnetic field, the photon and the second quantizaton

b. Road toward the Quantum Field Theory

1.4 The Road Toward the Quantum Field Theory

a. Early history

b. Infinities

c. Interactions and the gauge principle

d. Unsolved problems

2 Group Theory

2.1 Symmetries in Physics

a. Space-time symmetry

b. Permutation symmetry

c. Internal symmetries (gauge transformation)

2.2 Basics of Group Theory

a. Definition and examples of a group

b. More concepts

c. Linear transformation and related groups

2.3 Group Representation

a. Definition

b. Two dimensioal rotational group R_2 or SO(2)

2.4 Three Dimensioal Rotational Group R_3 or SO(3)

a. Parameterization

b. Generators

c. Irreducible representations

d. Example: particle in a central potential

2.5 The Lorentz Group and the Poincare Group

a. The Lorentz group

b. The Poincare group

c. Relativistic wave functions and field operators

3 Classical Field Theory

3.1 Lagrangian/Hamiltonian Mechanics and Field Theory

3.2 Noether's Theorem

a. Noether's theorem: formal derivation

b. Examples: space-time symmetry; internal symmetry

4 The Bosonic Fields

4.1 Canonical Quantization of Fields

4.2 The Klein-Gordon Field as Harmonic Oscillators

a. Free scalar field

b. Quantization relations

c. Particles and fields

4.3 The Propagation of the Klein-Gordon Field

a. Causality

b. Scalar field propagator

4.4 Vector Field Quantization

a. Massive vector field

b. Massless vector field and the photon

5 The Fermionic Fields

5.1 Lorentz Properties of the Dirac Equation

a. The Diract equation

b. Lorentz-invaraince of the Diract equation

c. The Weyl representation

5.2 Dirac Matrices and the Field Bilinears

a. Covariant bilinears

b. The chiral operator γ_5

5.3 Free-Field Solutions of the Dirac Equation

5.4 Quantization of The Dirac Field

a. Classical field

b. Canonical quantization

c. Anti-commutation relations

5.5 Discrete Symmetries

a. Parity

b. Time reversal

c. Charge conjugation

d. P, CP and CPT in nature

e. Majorana fermions and super-symmetry

6 Perturbation Theory for Interacting Fields

6.1 Interactions of Fields

a. Interactions and the perturbation method

b. Typical interactions for scalar, spinor and vector fields

6.2 Perturbation Expansion of Correlation Functions

a. The time-evolution operator

b. The state evolution

6.3 Wick's Theorem

6.4 Feynman Diagrams

a. The propagator

b. The interactions vertices

c. Feynman rules for scalars

d. Feynman rules for fermions

6.5 The Scattering Matrix Elements and the Cross sections

a. The S and T matrices

b. The scattering cross section

c. Feynman rules for the S-matrix: $i\mathcal{M}$ and Feynman diagrams

6.6 Phase Space and Kinematics

a. General phase space

b. Two body kinematics

7 Elementary Processes in QED

7.1 Non-relativistic Potential

a. The Coulomb potential

b. The Yukawa potential

7.2 Muon pair production $e^+e^- \to \mu^+\mu^-$

7.3 Quark pair and hadron production $e^+e^- \to q\bar{q}$

8 Radiative Corrections and Renormalization

(depending upon the time availability.)